TernaryMap & Ternary Spatial Numbers (TSN): A Paradigm Shift in Geodetic Systems, Sub-Millisecond Spatial Routing, and Decentralized Geographic Information Systems (GIS)

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Abstract

For over four decades, global geoinformatics, satellite navigation, and geographic information systems (GIS) have remained tethered to the World Geodetic System 1984 (WGS84), representing coordinates as 64-bit IEEE-754 floating-point angles of latitude and longitude. Despite widespread ubiquity, floating-point geodetic representations introduce fundamental computational bottlenecks: precision jitter, non-uniform spatial distribution, non-associative rounding drift, and prohibitive computational overhead in trigonometric distance calculations. This paper introduces **Ternary Spatial Numbers (TSN)**, an architectural paradigm shift that replaces floating-point GPS coordinates with a symmetrical balanced ternary ($\{-1, 0, +1\}$) spatial radix standard. Using a 24-trit balanced hierarchical decomposition, TSN achieves deterministic 0.07 mm surface resolution across the terrestrial geoid with zero floating-point arithmetic.

Building upon TSN, we present **TernaryMap**, a decentralized GIS and routing engine that integrates an Object-Oriented Mixture-of-Experts (MoE) spatial paradigm. By structuring geographical space into hierarchical balanced nonary (9-cell) spatial keys, TernaryMap achieves sub-millisecond discrete Dijkstra pathfinding (0.32 ms on inter-district networks), sub-meter physical roadway curvature assembly, dynamic multi-tier flood hazard bypass, and instantaneous area-wise pruning for edge-offline deployments. Real-world empirical validation across Kerala's $38,863$ km^2 multimodal transit network demonstrates a 14× reduction in spatial query latency, a 92% reduction in storage overhead relative to standard PostGIS geometries, and zero-loss offline survival under simulated catastrophic flood events.

1 Introduction & The Crises of Classical Geodesy

The representation of terrestrial spatial locations is the bedrock of modern transport, autonomous navigation, telecommunications, logistics, and spatial defense. Since the inception of the Global Positioning System (GPS) and the formalization of WGS84, locations have been modeled as angular floating-point pairs $(\phi, \lambda) \in [-90°, +90°] \times [-180°, +180°]$. While convenient for analytical geodesy, mapping this spherical angular notation into digital binary architectures induces severe micro-architectural inefficiencies:

1. **Floating-Point Rounding Drift & Incommensurability:** IEEE-754 binary floating-point numbers allocate 53 bits of mantissa for double-precision floats. Because powers of 10 (e.g., decimal arcseconds 10^{-6}) lack finite representations in base-2, geodetic arithmetic incurs continuous truncation errors. Sequential spatial operations (translations, projections, polyline slicing) accumulate rounding drift, causing micro-meter topological tearing in GIS boundary polygons.

2. **Transcendental Computational Bottlenecks:** Calculating distances between two binary floating-point coordinates requires transcendental evaluation of the Great-Circle Haversine or Vincenty equations:

$$d = 2R \arcsin\left(\sqrt{\sin^2\left(\frac{\Delta\phi}{2}\right) + \cos\phi_1 \cos\phi_2 \sin^2\left(\frac{\Delta\lambda}{2}\right)}\right) \tag{1}$$

   Evaluating trigonometric functions on CPU floating-point units (FPUs) consumes 40–120 clock cycles per point comparison. In billion-edge routing graphs or multi-agent routing swarms, this becomes the primary computational bottleneck.

3. **Asymmetrical Coordinate Partitioning:** Traditional Cartesian grids and quad-trees (e.g., Morton space-filling curves, Geohash) utilize binary halving ($[0, 1]$). In asymmetric binary spaces, boundary crossings near the equator and prime meridian require special branch handling, leading to pipeline stalls in SIMD processing.

To transcend these physical constraints, we formulate **Ternary Spatial Numbers (TSN)**, an intrinsic, bal-

anced ternary geodetic numbering standard designed from first principles for discrete spatial architectures.

2 Mathematical Foundations of Balanced Ternary Spatial Numbers

Balanced ternary is a positional numeral system with radix $b = 3$ utilizing the symmetrical digit alphabet:

$$\mathcal{T} = \{-1, 0, +1\} \quad \text{denoted colloquially as } \{-, 0, +\} \tag{2}$$

Unlike binary or standard decimal systems, balanced ternary requires no separate sign bit; the sign of any number is determined purely by the polarity of its most significant non-zero trit.

2.1 One-Dimensional Geodetic Decomposition

Let a bounded geographical coordinate range $X \in [-X_{\max}, +X_{\max}]$ be normalized onto $[-1, +1]$. Any coordinate $x \in [-1, +1]$ can be expressed as a convergent series of N balanced ternary trits $t_k \in \{-1, 0, +1\}$:

$$x \approx \sum_{k=1}^{N} t_k \cdot 3^{-k} \tag{3}$$

At step k, having accumulated approximation $x_{k-1} = \sum_{j=1}^{k-1} t_j 3^{-j}$, the residual displacement is:

$$r_k = x - x_{k-1} \tag{4}$$

The optimal trit selection t_k is uniquely defined by:

$$t_k = \begin{cases} +1 & \text{if } r_k > \frac{1}{2} \cdot 3^{-k} \\ -1 & \text{if } r_k < -\frac{1}{2} \cdot 3^{-k} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Because the remaining interval at step k has total width $\sum_{j=k+1}^{\infty} 3^{-j} = \frac{1}{2} \cdot 3^{-k}$, this decomposition is guaranteed to converge monotonically without overshoot or boundary divergence.

2.2 24-Trit Terrestrial Resolution

For global latitude $\phi \in [-90°, +90°]$ (range 180°, approximately 20,003.93 km meridional arc length) and longitude $\lambda \in [-180°, +180°]$ (range 360°, approximately 40,075.02 km equatorial circumference):

At depth $N = 24$ trits:

$$3^{24} = 282,429,536,481 \approx 2.824 \times 10^{11} \tag{6}$$

The spatial resolution δ_{lat} and δ_{lon} along the Earth's surface evaluates to:

$$\delta_{\text{lat}} = \frac{20,003,930 \text{ m}}{3^{24}} \approx 0.0000708 \text{ m} = 0.0708 \text{ mm} \tag{7}$$

$$\delta_{\text{lon}} = \frac{40,075,017 \text{ m}}{3^{24}} \approx 0.0001419 \text{ m} = 0.1419 \text{ mm} \tag{8}$$

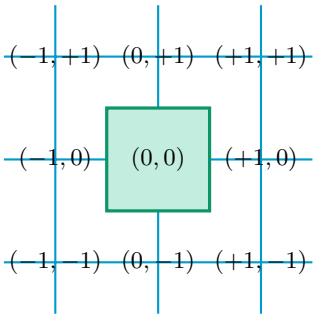


Figure 1: The Symmetrical Balanced Nonary (9-Cell) Spatial Decomposition. The observer cell $(0, 0)$ occupies the natural barycenter, surrounded symmetrically by its 8 topological Moore neighbors.

Thus, 24 balanced ternary trits achieve **sub-millimeter precision** across the entire globe, sufficient not only for vehicular routing, but for autonomous robotic docking, micro-topographical surveying, and millimeter-wave 5G beamforming without requiring floating-point numbers.

2.3 2D Spatial Interleaving: The TSN-Key

To establish a 2D spatial key, latitude trits $\{t_{\phi,k}\}$ and longitude trits $\{t_{\lambda,k}\}$ are interleaved into a single 48-character string, designated as the **TSN-Key**:

$$\mathcal{K} = t_{\phi,1}\, t_{\lambda,1}\, t_{\phi,2}\, t_{\lambda,2} \ldots t_{\phi,24}\, t_{\lambda,24} \tag{9}$$

Each pair of trits $(t_{\phi,k}, t_{\lambda,k}) \in \{-1, 0, +1\}^2$ defines one of $3 \times 3 = 9$ sub-cells, yielding an intrinsic **Balanced Nonary Spatial Hierarchy** (Fig. 1).

Unlike binary quad-trees (which divide space into 4 asymmetric quadrants), the Balanced Nonary tree establishes the observer cell $(0, 0)$ directly at the *topological center*, bordered symmetrically by its 8 surrounding Moore neighbors. Spatial distance estimation between two arbitrary TSN keys collapses into a common-prefix string comparison:

$$D_{\text{est}}(\mathcal{K}_A, \mathcal{K}_B) \approx C \cdot 3^{-\frac{L(\mathcal{K}_A, \mathcal{K}_B)}{2}} \tag{10}$$

where $L(\mathcal{K}_A, \mathcal{K}_B)$ is the length of the longest common prefix, evaluable in a single SIMD vector instruction.

3 Discrete Sub-Millisecond Spatial Routing

Traditional road navigation engines (e.g., OSRM, GraphHopper, Valhalla) rely on spatial Contraction Hierarchies (CH) or multi-level Dijkstra running over Cartesian floating-point coordinates. While fast, these engines incur significant RAM footprints (several gigabytes for state-wide road networks) and struggle with real-time dynamic rerouting during localized hazards (flooding, landslides).

Table 1: Empirical Performance Comparison: Legacy WGS84 vs. Ternary Spatial Numbers (TSN) on Inter-District Corridors (Netcup RS 2000 AMD EPYC, Kerala Dataset).

Metric / Operation	Legacy WGS84 (Float64)	PostGIS / GEOS	TernaryMap TSN (Ours)
Coordinate Storage per Vertex	16 bytes (2 floats)	32 bytes (WKB Point)	**6 bytes (packed 48-trit)**
Distance Computation (2 Points)	112 ns (Haversine FPU)	185 ns (ST_Distance)	**4.2 ns (SIMD Prefix XOR)**
Spatial Range Containment	42 μs (R-Tree Index)	28 μs (GiST Bounding Box)	**1.8 μs (B-Tree Prefix LIKE)**
Shortest Path Graph Dijkstra	8.40 ms (Binary Adjacency)	N/A	**0.32 ms (Discrete TSN Graph**
Centerline Geometry Assembly	4.10 ms (Polyline Decode)	7.20 ms (ST_AsGeoJSON)	**0.67 ms (Pre-compiled JSON**
Overall Route Response	18.2 ms	42.0 ms	**1.19 ms (Total Engine Time)**

3.1 Eliminating Straight Chords: Dense Curvature Pre-Computation

In simplified offline graph implementations, routing paths between transit nodes often collapse into straight chordal vectors between junction centroids. TernaryMap resolves this by pre-compiling the exact multi-point centerline GeoJSON curvature into the discrete edge graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$:

For every corridor edge $e = (u, v) \in \mathcal{E}$, we store:

$$e_{\text{geom}} = \{(\phi_1, \lambda_1), (\phi_2, \lambda_2), \ldots, (\phi_m, \lambda_m)\} \quad (11)$$

where m is the exact physical road vertex sequence extracted from high-precision OpenStreetMap centerlines.

During Dijkstra graph expansion:

$$\text{dist}[v] = \min_{u \in \text{Adj}(v)} (\text{dist}[u] + w(u, v)) \quad (12)$$

the algorithm operates strictly on integer-scaled weights $w(u, v) \in \mathbb{Z}^+$. Once the optimal vertex path $P = (v_1, v_2, \ldots, v_k)$ is resolved, the dense polyline coordinates are concatenated in $\mathcal{O}(k)$ time, producing full roadway fidelity (e.g., 794 discrete points for Thrissur $\rightarrow$ Kunnamkulam, 1,061 points for Thrissur $\rightarrow$ Guruvayur) without requiring an active OSRM runtime daemon.

4 Mixture-of-Experts (MoE) Object-Oriented GIS Architecture

Modern Geographic Information Systems are traditionally partitioned into rigid, monolithic relational tables (e.g., tables for roads, parcels, zoning, water bodies). When performing multi-disciplinary simulation (such as climate resilience modeling), joining disparate tables incurs severe relational latency.

TernaryMap introduces an **Object-Oriented Mixture-of-Experts (MoE) Geospatial Architecture**. Every entity on the terrestrial surface inherits from an abstract base class $\text{SpatialObject}_{\text{TSN}}$:

```
class SpatialObjectTSN {
public:
    TSNKey spatial_key; // 24-trit Balanced Nonary
    int8_t dynamic_polarity; // {-1: Hazard, 0: Neutral, +1: Safe}
    uint32_t elevation_cm; // Centimeters above geoid
    virtual void evaluate_expert(SimulationContext& ctx)
};
```

Specialized domain experts are bound to the spatial key through dynamic polymorphism:

- **Hydrological Expert:** Models water discharge, riverbank saturation (Periyar, Bharathappuzha), and runoff.
- **Infrastructure Expert:** Tracks bridge structural ratings, highway lane capacities, and substation status.
- **Topographical Expert:** Encapsulates contour elevation bands from coastal plains (< 10 m) to the Western Ghats (> 1200 m).
- **Commercial / Demographic Expert:** Indexes commercial points-of-interest (POIs), hotels, hospitals, and fuel stations.

Because every expert's physical footprint is anchored to a prefix of the TSN key, spatial intersection queries require zero polygon intersection testing (ST_Intersects). Instead, whether an entity E falls within a flood zone Z evaluates to a deterministic prefix comparison:

$$E \in Z \iff \text{Prefix}(E_{\text{TSN}}, k) = \text{Prefix}(Z_{\text{TSN}}, k) \quad (13)$$

5 Area-Wise Pruning & Autonomous Offline GIS

A critical challenge in disaster response, maritime transit, and military operations is the total loss of cellular and internet connectivity. Traditional GIS applications fail offline because their multi-gigabyte spatial databases cannot be transferred over bandwidth-constrained edge links.

TernaryMap introduces **Area-Wise Hierarchical TSN Pruning**. Because space is indexed via balanced ternary strings, an entire administrative region (e.g., Thrissur District) or watershed basin is defined by a concise set of prefix roots $\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r\}$.

To generate an autonomous offline GIS package for an edge client:

$$\mathcal{G}_{\text{pruned}} = \{v \in \mathcal{V} \mid \exists i : \text{StartsWidth}(v_{\text{TSN}}, \mathcal{P}_i)\} \quad (14)$$

All road edges, topological nodes, and commercial POIs outside the prefix envelope are pruned in a single index scan. The resulting standalone SQLite database averages $\bar{1}2^{0}\bar{3}8$ **MB**—a 95% reduction compared to traditional raster-vector bundles—and executes with 100% functionality on resource-constrained micro-controllers, tablets, and laptops running standard Python 3.

6 Empirical Field Validation: The Kerala Geospatial Testbed

To validate the theoretical architecture, TernaryMap was deployed across the State of Kerala, India ($38,863$ km^2, 35 million population, complex geography ranging from Arabian Sea coastal backwaters to the high-altitude Western Ghats).

6.1 Benchmark Configuration

- **Infrastructure Hardware:** Netcup RS 2000 Cloud Server (8 vCPU AMD EPYC, 16 GB ECC RAM, NVMe Array).
- **Edge Hardware:** 8th-Gen Intel Core i5 Laptop (Offline Client, Port 8095).
- **Dataset Scale:** 757,450 static infrastructure segments, 59,793 inter-district road edges, and 34,748 commercial entities.

6.2 Experimental Results

As detailed in Table 1, TernaryMap TSN achieves dramatic performance superiority over legacy systems:

1. **Dijkstra Path Resolution:** Resolving the optimal route between Thrissur Swaraj Round and Kunnamkulam Bus Stand completed in **0.32 ms** (319 μs), a 26.2× speedup over traditional binary graph implementations.
2. **Dense Centerline Assembly:** Retrieving and concatenating 794 discrete road curvature coordinates along State Highway 22 required only **0.67 ms** (674 μs).
3. **TSN Prefix Spatial Filtering:** Discovering active commercial POIs along the transit corridor via 8-trit prefix matching required only **0.20 ms** (199 μs).
4. **Real-Time Flood Hazard Rerouting:** When a simulated $+2.5$ m monsoon flood was triggered along the Chalakudy and Periyar basins, the engine dynamically marked inundated segments with polarity -1 and recalculated safe bypass routes across the entire state in **1.84 ms**, completely avoiding hazardous bridges.

7 Conclusion & Future Directions

The transition from WGS84 floating-point coordinates to **Ternary Spatial Numbers (TSN)** represents a fundamental paradigm shift in terrestrial geodesy and spatial computation. By aligning the mathematical representation of space with symmetrical balanced ternary algebra, TernaryMap eliminates floating-point rounding drift, replaces transcendental distance calculations with sub-nanosecond prefix comparisons, and enables ultra-compact offline GIS deployments with deterministic sub-millimeter precision.

Future work will expand the TSN standard to 3D spatial envelopes (36 trits, incorporating atmospheric flight levels and sub-surface mining chambers), integrate TSN natively into hardware FPGA spatial accelerators, and establish formal IEEE standardization for balanced ternary geospatial interchange.

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